Phantom Cosmic Dynamics: Tracking Attractor and Cosmic Doomsday

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We study the dynamics of phantom model and give the conditions for potentials to admit tracking attractor, de Sitter attractor and big rip attractor. Especially, we show that phantom models with exponential and inverse power law potentials do not admit the tracking attractor solution. The "tachyonic" stability of the system at/near the attractors as well as the quantum stabiltiy have been studied.

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I. INTRODUCTION

Astronomical observations on the CMB anisotropy [1, 2, 3], the relation between red-shift and luminosity distance of Supernovae [4, 5, 6] as well as the galaxy distribution, clumping and spreading [7] depicted that our Universe is spatially flat, with about two-thirds of energy density resulted from dark energy that has an equation of state $w < -1/3$ and accelerates the expansion of the Universe. The nature of this substance is quite unusual and there is no justification for assuming it resemble known forms of matter or energy. Candidates for dark energy have been widely studied and focus on a dynamically evolving scalar field (quintessence [8, 9, 10, 11], with $w > -1$ and phantom [12], with $w < -1$) and cosmological constant. Present observation data constrain the the range of the equation of state of dark energy as $-1.38 < w < -0.82$ [13], which indicates the possibility of dark energy with $w < -1$, debuted as Phantom or super quintessence [12]. The realization of $w < -1$ could not be achieved by scalar field with positive kinetic energy and thus the negative kinetic energy is introduced although it violates some well known energy conditions [14]. Another important consequence of Phantom is the Big rip [15] or Big smash [16] phase, in which the scale factor of the Universe goes to infinity at a finite cosmological time. The cosmological implications of Phantom have been widely studied[17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] , the Phantom model with Born-Infeld type Lagrangian has been proposed [34] and its generalization to brane world has been done in Ref. [35].

One of the most important issues for dark energy models is the fine tuning problem and a good model should limit the fine tuning as much as possible. The dynamical attractor of the cosmological system has been employed to make the late time behaviors of the model insensitive to the initial condition of the field and thus alleviates the fine tuning problem. In quintessence models, the dynamical system has tracking attractor that makes the quintessence evolve by tracking the equation of state of the background cosmological fluid so as to alleviating the fine tuning problem[36, 37]. While in phantom models with canonical lagrangian, the kinetic energy term becomes negative, which make the phase space of the cosmological system behaves rather differently from those of the quintessence. The major difference is that the tracking attractor does not exist in the phantom system with exponential and inverse power law potentials, which admit tracking attractor solution in quintessence models. Accordingly, We give the condition for the phantom system to admit tracking attractor. On the other hand, there are also two late time attractors in the phantom system corresponding to the big rip phase[39] and de Sitter phase[40]. One cannot say, as a priori, whether our Universe will end with the big rip or with the de Sitter phase, therefore it will be interesting to study dynamical evolution of the Phantom models in general potentials and the conditions for big rip and de Sitter phase respectively.

II. BIG RIP AND DE SITTER LATE TIME ATTRACTORS

Since current observations favor flat Universe, we will work in the spatially flat Robertson-Walker metric. The corresponding equations of motion and Einstein equations could be written as,

$$\begin{aligned}
&\dot{H} = -\frac{\kappa^2}{2}(\rho_\gamma + p_\gamma - \dot{\phi}^2)\\
&\dot{\rho_\gamma} = -3H(\rho_\gamma + p_\gamma)\\
&\ddot{\phi} + 3H\dot{\phi} - V'(\phi) = 0\\
&H^2 = \frac{\kappa^2}{3}(\rho_\gamma + \rho_\phi)
\end{aligned} \tag{1}$$

where $\kappa^2 = 8\pi G$, ρ_γ is the density of fluid with a barotropic equation of state $p_\gamma = (\gamma-1)\rho_\gamma$, where $0 \le \gamma \le 2$ is a constant that relates to the equation of state by $w = \gamma - 1$; The over dot represents derivative with respect to t, the prime denotes derivative with respect to ϕ. $\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi)$ and $p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi)$ are the

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energy density and pressure of the ϕ field respectively, and H is Hubble parameter.

However, for an arbitrary potential, and considering the presence of other energy density, one cannot find analytically solvable models. Thus, we need to analyze the models via phase space analysis. Similar as in Ref.[36, 37, 38], we introduce the following dimensionless variables $x = \frac{\kappa}{\sqrt{6}H}\dot{\phi}$, $y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$, $\lambda = -\frac{V'(\phi)}{\kappa V(\phi)}$, $\Gamma = \frac{V(\phi)V''(\phi)}{V'^2(\phi)}$ and $N = \log a$. Then, the equation system(1) could be reexpressed as the following system of equations:

$$
\begin{aligned}
\frac{dx}{dN} &= \frac{3}{2}x[\gamma(1+x^2-y^2)-2x^2]-(3x+\sqrt{\frac{3}{2}}\lambda y^2) \\
\frac{dy}{dN} &= \frac{3}{2}y[\gamma(1+x^2-y^2)-2x^2]-\sqrt{\frac{3}{2}}\lambda xy \\
\frac{d\lambda}{dN} &= -\sqrt{6}\lambda^2 x(\Gamma-1)
\end{aligned}
\tag{2}
$$

Also, we have a constraint equation

$$
\Omega_\phi + \frac{\kappa^2\rho_\gamma}{3H^2} = y^2 - x^2 + \frac{\kappa^2\rho_\gamma}{3H^2} = 1 \tag{3}
$$

The equation of state for the scalar fields could be expressed in term of the new variables as

$$
w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{x^2+y^2}{x^2-y^2} \tag{4}
$$

Different from the case in exponential potential [39], the parameters λ and Γ here are variables dependent on ϕ. Thus, strictly speaking, the above system is not an autonomous system. So, just as was done in Refs. [36, 37], if we want to discuss the phase plane, we need certain constraints on the potential, or equivalently the condition under which the potential has the property we want, in order that we can get some explicit results.

In the following, we will read out the big rip attractor as well as de Sitter attractor from Eqs.(2) and specify the corresponding conditions for the potential. The big rip attractor is the dynamical attractor that corresponds to the Phantom domination $\Omega_\phi = 1$ and an equation of state $w < -1$. While, the de Sitter attractor also corresponds to Phantom domination but with $w = -1$. Looking at Eqs.(2), one can observe that physically meaningful critical points (x_c, y_c, λ_c) of the system are (Note that here we are only interested in the expansionist solutions $(y > 0)$)
(i). $(-\frac{\lambda_c}{\sqrt{6}}, \sqrt{1+\frac{\lambda_c^2}{6}}, \lambda_c)$ with $\Gamma = 1$ and $\lambda_c \neq 0$
(ii). $(0, 1, 0)$ for all Γ.
(iii). $(0, 0, \lambda_c)$ for all Γ.
where λ_c could be fixed by $\Gamma = 1$. To gain some insight into the property of the critical points, we write the variables near the critical points (x_c, y_c, λ_c) in the form $x = x_c + u$, $y = y_c + v$, and $\lambda = \lambda_c + \delta$ with u, v, δ the perturbations of the variables near the critical points. Substitute the expression into the system of equations (2), one can obtain the equations for the perturbations. The coefficients of the perturbation equations form a 3×3 matrix M whose eigenvalues determine the type and stability of the critical points.

(i) Big rip attractor: in the above case (i), when $\Gamma \simeq 1$, from Eq.(2), we know that λ is almost constant. So, similar as was done in Refs. [36, 37], we consider only the first two equations of Eqs.(2). Thus, the corresponding matrix M should be two dimensional and the corresponding eigenvalues are $(-3 - \frac{\lambda_c^2}{2}, -3\gamma - \lambda_c^2)$, which indicate that this critical point is a dynamical attractor of the system. It is not difficult to evaluate that at the critical point, the cosmic energy density parameter $\Omega_\phi = 1$ and the equation of state $w = -1 - \frac{\lambda_c^2}{3}$. So, this attractor corresponds to an equation of state $w < -1$ and will lead to the catastrophic big rip. It is worth noting that for an arbitrary potential, λ_c is not a constant, which could be determined by solving $\Gamma = 1$ for a given potential. While in the exponential potential, λ_c is denoted as λ and is a constant parameter of the potential. For a general potential $V(\phi)$, the condition for the existence of the above mentioned big rip attractor is $\Gamma = 1$(for practical purpose, this condition could be $\Gamma \simeq 1$, as was done in Refs. [36, 37]) and $\lambda \neq 0$.

(ii) de Sitter attractor: in the above case (ii), the corresponding eigenvalues of matrix M are $(-3, 0, -3\gamma)$, which indicate that the critical point is a dynamical attractor corresponding to equation of state $w = -1$ and cosmic energy density parameter $\Omega_\phi = 1$, which is a de Sitter attractor[40]. The condition of such a de Sitter attractor is $\lambda = 0$. According to the previous definition for λ, this condition is equivalent to that the attractor appears at the extremum of the potential. We must point out that when keeping the perturbation to the linear order, the right hand side of the third equation in Eqs.(2) will vanish. Thus the condition $\lambda = 0$ is only a less restrictive one, from which we cannot determine if the extremum is maximum or minimum. To extract more information, one needs to either keep higher order perturbation or choose different dimensionless variables as was done in Ref. [40], in which we choose $x = \frac{\phi}{\phi_0}$, $y = \frac{\dot{\phi}}{\phi_0^2}$ and reduce Eqs.(1) to a different form. The analysis indicates that the extremum must be a maximum for the existence of de Sitter attractor for Phantom models. For details, please refer to Ref. [40].

In the above case (iii), the critical point corresponds to $\Omega_\phi = 0$ and the corresponding definition of w_ϕ becomes meaningless. Moreover, the corresponding eigenvalues of the M matrix is $(0, 3(\gamma-2)/2, 3\gamma/2)$, which indicates the critical point is not a dynamical attractor. So, we won't address it into details.

Next, let's consider two specific models that admit big rip attractor and de Sitter attractor respectively. To do so, we must specify the potential. For the big rip attractor, we have the requirement that $\lambda \neq 0$ and $\Gamma = 1$

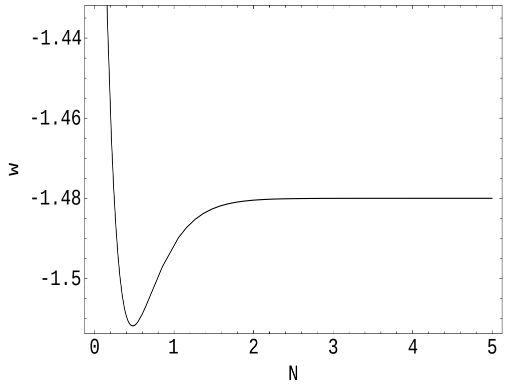

FIG. 1: The evolution of equation of state w vs. N. We choose dimensionless parameter $\frac{\alpha}{\kappa} = 1.2$ and the $\gamma = 1$.

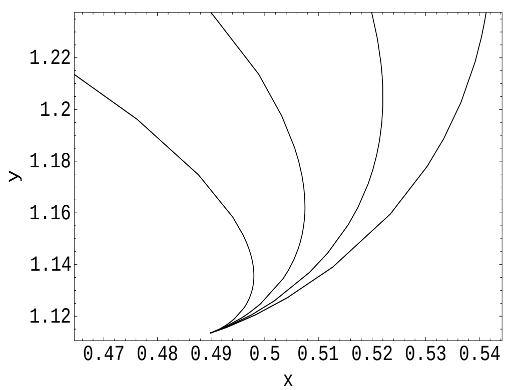

FIG. 2: Phase graph of the model for different initial x and y. We choose dimensionless parameter $\frac{\alpha}{\kappa} = 1.2$ and the $\gamma = 1$.

(or $\Gamma \simeq 1$ as approximation for practical purpose). It won't be difficult to find that the solution for the exact $\Gamma = 1$ would be that $V(\phi) = C_1 e^{C_2\phi}$ with C_1 and C_2 as arbitrary constants. Conventional study for quintessence models focus on the case that $C_2 < 0$. Here, C_2 could be both positive and negative constant. For definite, we choose $V(\phi) = V_0 e^{\alpha\phi}$ with α a positive constant, and thus $\lambda = -\frac{\alpha}{\kappa}$. In Fig.1 and Fig.2, we plot the numerical results.

For the de Sitter attractor, we require $\lambda = 0$, which is equivalent to that the critical point is the extremum of the potential. We choose the potential

$$V(\phi) = V_0 - \sigma_0(\frac{\phi}{\phi_0})^2 \tag{5}$$

as a toy model, in which the de Sitter attractor corresponds to the maximum of the potential Eq.(5). The corresponding numerical results listed in Fig.3 and Fig.4

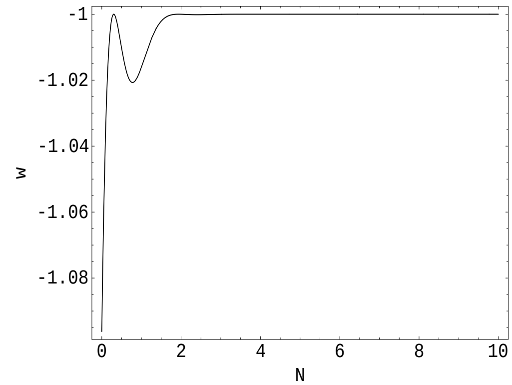

FIG. 3: The evolution of equation of state w vs. N, in which for simplicity, we choose $V_0 = \phi_0 = \kappa = \sigma_0 = \gamma = 1$

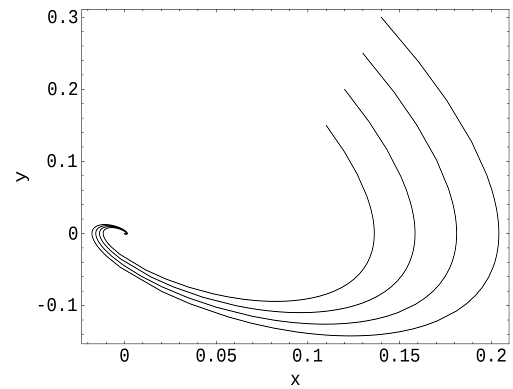

FIG. 4: The phase graph of the system with $x = \phi/\phi_0$ and $y = \dot{\phi}/\phi_0^2$, which are so defined for convenience of evaluation. The condition for evaluation is $V_0 = \phi_0 = \kappa = \sigma_0 = \gamma = 1$.

III. TRACKING ATTRACTOR

In this section, we will show the tracking attractor behavior of the dynamical system that are not manifested in the above analysis. We consider the situation that λ is very large and Γ is not 1 but nearly constant. It will be convenient for the discussion if we make the following transformation $\epsilon = \frac{1}{\lambda}$, $x = \epsilon X$ and $y = \epsilon Y$[37]. Then the dynamical system could be rewritten in terms of the new variables X and Y as:

$$\begin{aligned}\frac{dX}{dN} &= -\sqrt{6}(\Gamma-1)X^2 + \frac{3}{2}\gamma X - 3X - \sqrt{\frac{3}{2}}Y^2 \\ \frac{dY}{dN} &= -\sqrt{6}(\Gamma-1)XY - \sqrt{\frac{3}{2}}XY + \frac{3}{2}\gamma Y\end{aligned} \tag{6}$$

where we have omitted the terms contain ϵ. The critical points of the above autonomous system Eq.(6) corresponding to the tracking solution is $(X_c, Y_c) = (\sqrt{\frac{3}{2}}\gamma_e, \sqrt{\frac{3}{2}\gamma_e(\gamma_e-2)})$, where $\gamma_e \equiv \frac{\gamma}{2\Gamma-1}$ is a defined effective barotropic index of the scalar field. The corresponding cosmic energy density parameter for phantom is $\Omega_\phi = -\frac{3\gamma_e}{\lambda^2}$ and the equation of state for phan-

tom is $w_\phi = w_e$ with $w_e \equiv \gamma_e - 1$. Clearly, to make Ω_ϕ physically meaningful, we need $\Gamma < 1/2$ and therefore $\gamma_e < 0$. Since λ is large, then Ω_ϕ is small and as a consequence, the background dominates. Also, at this point, the energy density parameter of the phantom field will track the effective barotropic index of the scalar field γ_e, by which the name of tracking solution is justified. It is not difficult to observe that although the equation of state of the background cosmological fluid is positive (0 for matter and 1/3 for radiation), the effective equation of state could be less than -1, which is required for the the cosmic density parameter for phantom Ω_ϕ to make sense. This tracking property is very different from that of quintessence models in which the corresponding critical point is $(X_c, Y_c) = (\sqrt{\frac{3}{2}}\gamma_e, \sqrt{\frac{3}{2}\gamma_e(2-\gamma_e)})$ with the γ_e defined the same as that in the above phantom case. But the corresponding cosmic density parameter is $\Omega_{quint} = \frac{3\gamma_e}{\lambda^2}$ which is just the opposite of the expression for phantom. This sign difference lead to different requirements for Γ, that is, for quintessence, Γ must be greater than 1/2 while for phantom Γ must be less than 1/2 so as to make their respective cosmic density parameters physically meaningful. This constraint on the potentials for phantom excludes the exponential potential($V(\phi) = V_0 e^{\alpha\phi}$ with $\Gamma = 1$)and inverse power law potential($V(\phi) = A/\phi^n$ with $\Gamma = 1 + \frac{1}{n}$) that have been used in quintessence models. A simple potential that satisfies the above condition takes the form $V(\phi) = A\phi^{1+\frac{1}{n}}$ with $n > 1$. In this case, $\Gamma = \frac{1}{1+n}$.

Next, let's show the stability of the critical point. To do so, we use the similar approach used in previous section and find out the eigenvalues of the perturbation equation of the dynamical system near the critical point. We write the $X = X_c + U$ and $Y = Y_c + V$, with U, V the perturbations. Then the equation system could be written as, up to the first order of the perturbations,

$$\begin{aligned} \frac{dU}{dN} &= [-3 + (9/2 - 3\Gamma)\gamma_e]U - 3\sqrt{\gamma_e(\gamma_e - 2)}V \\ \frac{dV}{dN} &= -3/2(2\Gamma - 1)\sqrt{\gamma_e(\gamma_e - 2)}U \end{aligned} \quad (7)$$

The eigenvalues are

$$\begin{aligned} \lambda_1 &= -\frac{3}{4}\Big[2 + (2\Gamma - 3)\gamma_e \\ &\quad + \sqrt{4 + (4 - 24\Gamma)\gamma_e + (1 + 2\Gamma)^2\gamma_e^2}\Big] \\ \lambda_2 &= -\frac{3}{4}\Big[2 + (2\Gamma - 3)\gamma_e \\ &\quad - \sqrt{4 + (4 - 24\Gamma)\gamma_e + (1 + 2\Gamma)^2\gamma_e^2}\Big] \end{aligned} \quad (8)$$

According to our previous discussion, we know that $\Gamma < 1/2$ is required for the solution physically meaningful. Closely examine the two eigenvalues, one can find that the first one always has a negative real part. While for the second one, the term in the square root become negative if $\Gamma < \frac{5-4\sqrt{2}}{14}$when $\gamma = 1$(matter dominated epoch) and $\Gamma < \frac{19-8\sqrt{6}}{46}$ when $\gamma = 4/3$ (radiation dominated epoch). In these two cases, the critical point is stable. While for the range of $\frac{5-4\sqrt{2}}{14} \leq \Gamma < 1/2$ and $\frac{19-8\sqrt{6}}{46} \leq \Gamma < 1/2$ corresponding to the matter and radiation dominated epoch respectively, we plot the second eigenvalue vs. Γ in Fig.5 and Fig. 6. From the plots, one can see that both cases yield negative eigenvalues and therefore the critical point is stable for the range $\Gamma < 1/2$. That is, the tracking solution is a dynamical attractor of the system.

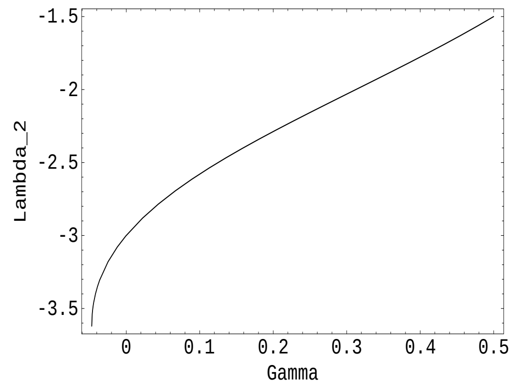

FIG. 5: The second eigenvalue λ_2 vs. Γ at the matter dominated epoch.

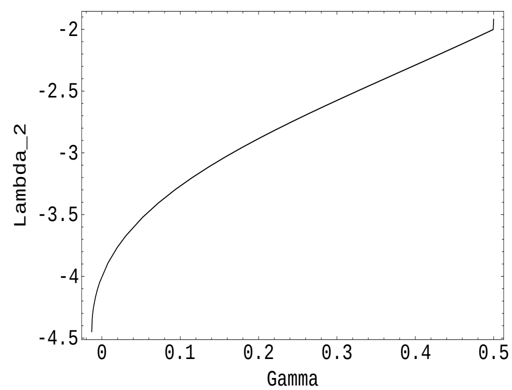

FIG. 6: The second eigenvalue λ_2 vs. Γ at the radiation dominated epoch.

IV. TACHYONIC STABILITY NEAR THE ATTRACTORS

In this section, we will study the "tachyonic" instability[1] of the dynamical system near the dynamical attrac-

[1] We call the system is tachyonically instable if the effective mass of the perturbation becomes imaginary

tors. The reason for us to discuss the stability only near the attractors is that we consider that our universe is just at or near one of the attractors. The stability of the system at the attractors does not necessarily guarantee the stability during its evolution, but if we took a anthropic philosophy and the above assumption, we can say that we are just living in a universe where the phantom evolves safely to its stable attractors. In synchronous gauge, the perturbed equation of motion reads [12, 14]

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + (k^2 - V''(\phi)) = -\frac{1}{2}\dot{h}\dot{\phi} \tag{9}$$

where the h is the trace of h_{ij} (the metric perturbation) and $\delta\phi_k$ is a fourier mode of the phantom field perturbation. The prime denotes the derivative with respect to ϕ and the dot denotes the derivative with respect to t. The effective mass for the perturbation is $[k^2 - V''(\phi)]^{1/2}$. For the potentials with negative $V''(\phi)$, it will be always stable. While for potentials with positive V'',the stability requires that the wave number of the perturbation should be $k < k_{crit} = \sqrt{V''(\phi)}$. In terms of the dimensionless variables we introduced previously, the critical wave number could be written as

$$k_{crit} = \sqrt{V''(\phi)} = \sqrt{3}Hy|\lambda|\sqrt{\Gamma} \tag{10}$$

At the tracking attractor, we can easily obtain that $y_c = \sqrt{\frac{3\gamma_e(\gamma_e-2)}{2\lambda_c^2}}$ and the corresponding critical wave number will be $k_{crit} = 3H\sqrt{\frac{\Gamma\gamma_e(\gamma_e-2)}{2}}$. If we choose the Hubble parameter H as the present value and γ_e as well as Γ are of order 1, then the stability will appear if the wave length of the perturbation is greater than about 10^{28} cm, which is even greater than the radius of our observable Universe. That is to say, the tracking attractor solution corresponds to a state that is "tachyonically" stable. At the big rip attractor, the critical wave number should be $k_{crit} = |\lambda_c|H\sqrt{3 + \frac{\lambda_c^2}{2}}$, with λ_c a dimensionless variable that can be specified by solving $\Gamma = 1$. If we also choose the Hubble parameter H as the present value and λ_c is of order 1, then the situation will be similar to the above case for tracking attractor and the model is stable for practical purpose at the big rip attractor. At the de Sitter attractor, we have $V''(\phi_{crit}) < 0$ so that the point corresponds to the maximum of the potential [40]. Therefore, the models will always be stable at the de Sitter attractor. From above discussion, we know that the de Sitter phase, big rip phase and tracking phase all could not only be dynamical attractors under properly constrained potentials but also stable state in terms of the tachyonic stability. However, as we have mentioned at the beginning of this section, the stability at the attractors does not guarantee its evolution. But this could be circumvented by adopting an anthropic view.

V. QUANTUM STABILITY

Besides the tachyonic instability we have analyzed in section IV, there is also another instability, quantum instability, that plagued phantom models[14]. Since dark energy is assumed to be weakly interacted with other matter except via gravity, it would be natural to consider only the phantom decay relevant to gravitons, i.e. a phantom decays into other phantoms and gravitons. Clearly, the simplest process will be one phantom decays into two other phantoms and a graviton as

$$\phi_i \rightarrow h + \phi_1 + \phi_2 \tag{11}$$

Follow the discussions in Ref. [14], we firstly consider the decay in the tree level with potential coupling and calculate the corresponding λ_{eff} for the potentials in this paper, i.e. exponential potential $V(\phi) = V_0 e^{\alpha\phi}$, quadratic potential Eq .(5) and an analytically solvable potential(its analysis will be presented in section VI).

$$V(\phi) = V_0\{\sin[\sqrt{3}\kappa(\phi - \phi_0)] + \csc[\sqrt{3}\kappa(\phi - \phi_0)]\} \tag{12}$$

It is not difficult to evaluate that the corresponding λ_{eff} are

$$\lambda_{eff} = \begin{cases} \frac{\alpha^3 V(\phi_0)}{6M_p}, & \text{for exponential potential} \\ 0 \quad\quad\;\; , & \text{for quadratic potential} \\ \frac{50V_0\kappa^3}{40\sqrt{3}M_p}, & \text{for potential Eq.(12)} \end{cases} \tag{13}$$

Similar as in Ref.[14], ϕ_0 is of the order of M_p. Note that the parameters in the potentials are of the order $\alpha \sim \kappa \sim 1/M_p$, the λ_{eff} for exponential potential as well as the solvable potential (12) are both of the order $V_0/M_p^4 \sim 10^{-120}$, the same as that in Ref.[14] and thus need the same momentum cutoff. But for the quadratic potential, $\lambda_{eff} = 0$ and therefore the corresponding model need no momentum cutoff and is stable in the tree level potential coupling. As we have clarified at the beginning of this section, the decay described above is the simplest one and more complicated decay process will need higher derivative of the potentials. So, the quadratic potential model is stable if we consider only the tree level potential coupling. But the other two potentials need momentum cutoff of $\Lambda < 10^{60}M_p$, which is still acceptable[14].

While, if we also consider the derivative coupling, which is potential independent, the situation will be the same as that in Ref.[14] and the optimistic estimation of the momentum cutoff for the models should be less than $10^{20}M_p \sim 100$MeV. However, this is not necessarily impossible for phantom if we assume the scalar field theory is only valid up to relatively low momenta or find some mechanism to suppress derivative couplings, leaving only the couplings of gravitons to the potential, which were consistent with a cutoff as high as the Planck scale[14].

The point is that if the observations eventually do suggest dark energy with an equation of state $w < -1$ and if it is modelled by phantom, then it must be stable with some mechanisms such as those described above.

VI. TWO ANALYTICAL MODELS

In the following, to expose the above discussion more specifically, we discuss two solvable Phantom models that correspond to quasi-de Sitter and big rip late time phase respectively. We consider only the case that Phantom become dominant. The potential for the model corresponding to quasi-de Sitter solution is in the periodic potential Eq.(12). For simplicity, we consider only the behavior of $V(\phi)$ for $\phi \in [\phi_0, \phi_0 + \frac{\pi}{\sqrt{3}\kappa}]$. The quasi-de Sitter solutions are

$$\begin{aligned} a &= a_0\{\sec[\sqrt{3}\kappa(\phi-\phi_0)]\}^{1/3} \qquad (14)\\ t &= t_0 + \frac{1}{(288V_0^2)^{\frac{1}{4}}\kappa}\{\ln\left(\frac{1+[1-(\frac{a_0}{a})^6]^{\frac{1}{4}}}{1-[1-(\frac{a_0}{a})^6]^{\frac{1}{4}}}\right) \\ &\quad +2\arctan[1-(\frac{a_0}{a})^6]^{\frac{1}{4}}\} \\ \rho_\phi &= \sqrt{2}V_0[1-(\frac{a_0}{a})^6] \\ w &= -\frac{1}{1-(a_0/a)^6} \end{aligned}$$

It is easily found that the scale factor $a(t)$ tends to infinite as $\phi \to \phi_0 + \frac{\pi}{2\sqrt{3}\kappa}$. Furthermore, as the Universe expands, the Phantom energy density ρ_ϕ tends to $\sqrt{2}V_0$ and the equation of state w_ϕ approaches -1.

The second potential for the model admitting big rip solution is

$$V(\phi) = V_0\exp[\sqrt{3}\kappa A(\phi-\phi_0)] \qquad (15)$$

where V_0 and A are positive constant. The solutions of the system are

$$\begin{aligned} a &= a_0\exp[\frac{\kappa}{\sqrt{3}A}(\phi-\phi_0)] \qquad (16)\\ t &= t_0 + \frac{2(2+A^2)^{1/2}}{\sqrt{3}\kappa(2V_0)^{1/2}A^2}[1-(\frac{a_0}{a})^{3A^2/2}] \\ \rho_\phi &= \frac{2V_0}{2+A^2}(\frac{a}{a_0})^{3A^2} \equiv \rho_0(\frac{a}{a_0})^{3A^2} \\ w_\phi &= -(1+A^2) \end{aligned}$$

Using Eq.(16), we find that the remaining time $t_r = t_{end} - t_0$ before the end of the Universe is given analytically by

$$t_r = \frac{2}{\sqrt{3\rho_0}\kappa A^2} \qquad (17)$$

In Eq.(17), putting in $\kappa\sqrt{\rho_0} = \sqrt{3}\Omega_\phi H_0$ with $\Omega_\phi = 0.73$ and $H_0^{-1} = 13.7Gyr$, one finds $t_r = 28.01$, 53.4 and 106.9 Gyr. for $w = -1.38$, -1.20 and -1.10 respectively.

VII. DISCUSSION

We study the phase space of phantom model in an arbitrary potential and give the corresponding conditions for tracking attractor, big rip attractor and de Sitter attractor. The big rip attractor and de Sitter attractor are two late time attractors that will determine the type of the cosmic doomsday. For the de Sitter attractor, we have $\lambda = 0$ and therefore a divergent Γ according to its definition. However, in the equations of motion Eqs.(2), such singularity will not appear because of the term $\lambda^2\Gamma = \frac{V''}{\kappa^2}$ gives regular value. On the other hand, the tracking attractor serves a similar function as it does in traditional quintessence models and could alleviate the fine tuning of the model. The observational constraint $-1.38 < w < 0.82$ is often interpreted as an observational support for the existence of Phantom in our Universe. We show that there maybe two possible phantom cosmic doomsday, big rip and de Sitter, which are both attractor phases of the dynamical system and compatible with current observations. At least before the $w < -1$ dark energy is excluded by observation completely, the phantom models will remain an interesting alternative.

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